Optimal Capacitor Placement and Sizing in Unbalanced Distribution Systems With Harmonics Consideration Using Particle Swarm Optimization

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***Abstract*—Shunt capacitors installation in distribution systems requires optimal placement and sizing. More harmonics are being injected into distribution systems. Adding shunt capacitors may lead to high distortion levels. The capacitor placement and sizing problem is a nonlinear integer optimization problem, with locations and ratings of shunt capacitors being discrete values. The goal is to minimize the overall cost of the total real power loss and that of shunt capacitors while satisfying operating and power quality constraints. This paper proposes to solve the problem using particle swarm optimization (PSO). A discrete version of PSO is combined with a radial distribution power flow algorithm (RDPF) to form a hybrid PSO algorithm (HPSO). The former is employed as a global optimizer to find the global optimal solution, while the latter is used to calculate the objective function and to verify bus voltage limits. To include the presence of harmonics, the developed HPSO was integrated with a harmonic power flow algorithm (HPF). The proposed (HPSO-HPF)-based approach is tested on an IEEE 13-bus radial distribution system (13-Bus-RDS). The findings clearly demonstrate the necessity of including harmonics in optimal capacitor placement and sizing to avoid any possible problems associated with harmonics.**

***Index Terms*—Harmonics, particle swarm, shunt capacitors.**

I. Introduction

SHUNT capacitors are commonly used in distribution systems to reduce power losses, improve voltage profile, and release system capacity. The achievement of such benefits among other benefits depends greatly on how optimally these shunt capacitors are installed. Studies have indicated that approximately 13% of generated power is consumed as loss at the distribution level. In addition, with the application of loads, the voltage profile tends to drop along distribution feeders below acceptable operating limits. Along with power losses and voltage drops, the increasing growth in electricity demand requires upgrading the infrastructure of distribution systems. Shunt capacitors can be of great help in enhancing the performance of distribution systems. Distribution systems are inherently unbalanced for several reasons. First, distribution systems supply single and three-phase loads through distribution transformers. Second, the phases of transmission lines are unequally loaded. Third, unlike those in transmission systems overhead lines in distribution systems are not transposed.

Due to the widespread use of harmonic-producing equipment in distribution systems, harmonics are propagated throughout those systems. Harmonics are undesirable and cause equipment overheating due to the excessive losses and potential malfunctioning of electric equipment. Inclusion of shunt capacitors without considering the presence of harmonic sources in the system may lead to an increase in harmonic distortion levels due to resonance between capacitors and the various inductive elements in the system.

Baghzouz developed a local variations-based heuristic approach to find the global optimal ratings of shunt capacitors such that the cost of total real power loss and that of shunt capacitors were minimized [1]. The optimal capacitor sizing problem was formulated as a nonlinear integer programming problem with inequality constraints. The constraints considered were the rms values of bus voltages and total harmonic distortions. The only harmonic source assumed was the utility substation. A heuristic algorithm based on local variations was proposed to overcome the prohibitive computational time associated with considering every single potential capacitor size at a given iteration. Yan accounted for the presence of harmonic-producing loads in distribution systems [2]. A hybrid differential evolution algorithm was developed to optimally locate and rate shunt capacitors in distorted distribution systems. A sensitivity test was done prior to the optimization process to determine the candidate buses for reactive power compensation. The objective was to minimize the cost of real power losses and that of shunt capacitors while satisfying some practical constraints. The results indicated that neglecting the presence of harmonic sources could cause a severe harmonic distortion problem. Carpinelli *et al.* solved the capacitor placement and sizing problem in a way that the overall cost was minimized [3]. The cost function involved the cost of real power losses, shunt capacitors, and harmonic distortions. An approximate power flow method and a linear harmonic power flow method were used to calculate the cost function at the fundamental and various harmonic frequencies.

Another optimization technique used to solve the optimal capacitor placement and sizing problem is genetic algorithms (GA). Abou-Ghazala proposed a GA to find the best combination of locations and ratings of shunt capacitors such that the total net savings were maximized [4]. Loss reduction was achieved through the proper installation of shunt capacitors

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while rms values of bus voltages and total harmonic distortions being kept within allowable limits. Nikham *et al.* also used a genetic algorithm to solve the optimal capacitor allocation and sizing problem taking the presence of harmonic sources into account [5]. The objective function consisted of the cost of real power losses and that of shunt capacitors to be installed. The cost associated with the reactive power injection was fixed for all possible capacitor sizes. In other words, the cost of the reactive power injected was assumed to be constant independent of the capacitor size. Masoum *et al.* developed a hybrid tool based on maximum sensitivity selection (MSS) and local variations (LV) to solve the optimal capacitor placement and sizing problem [6]. The former was used to enhance the convergence speed by narrowing down the search space, while the latter was employed to find the global optimal solution. Three harmonic distortion levels were considered for the system investigated. The system under investigation involved only one harmonic source and that was a six-plus converter. The results of the hybrid MSS-LV algorithm were compared with those of the MSS-based algorithm. In later work, Masoum *et al.* applied a fuzzy logic-based algorithm to solve the same problem [7]. Both the objective function and the constraints were fuzzified. Alpha cuts were used to direct the search process and to ensure that the objective function improved each time. The candidate buses were determined according to the objective function, constraints and reactive power compensation sensitivities. Two harmonic distortion levels were considered this time to compare the results obtained with those obtained by the MSS-based algorithm. A conclusion was drawn that the appropriate locations and ratings of shunt capacitors would not only improve voltage profiles but also would reduce harmonic distortion levels. Masousm *et al.* took advantage of the capability of genetic algorithms (GAs) to escape local optima [8]. Improvements in voltage profiles and power quality were achieved through the proper installation of fixed shunt capacitors in distorted distribution systems. The applicability of GA-based approach was proven to yield to better results compared to the previous work done by the same authors.

Another method based on particle swarm optimization (PSO) was offered in [9] to solve the capacitor placement and sizing problem considering harmonics. The problem was mathematically modeled as a nonconvex optimization problem. The objective function was augmented by quadratic penalty functions to account for inequality constraints. That is, the objective function was penalized whenever the inequality constraints were violated. The proposed PSO algorithm did not account for unbalanced operating conditions. Khalil *et al.* [10] proposed a binary PSO algorithm to find the best locations and ratings of fixed shunt capacitors in balanced distribution systems. The only harmonic source considered was the substation voltage. Their objective was to properly place and size shunt capacitors while keeping the cost of real power losses and that of shunt capacitors at a minimum. The objective function was subject to equality and inequality constraints.

II. Problem Formulation

The optimal capacitor placement and sizing problem is formulated as a constrained nonlinear integer optimization problem with both locations and sizes of shunt capacitors being discrete. The objective function encompasses the total cost of the total real power loss and that of shunt capacitors. The objective function is restricted by equality and inequality constraints.

Objective Function: The goal is to minimize the cost of the total real power loss and that of the shunt capacitor installation. The cost function is given by

$$F = K_p Ploss + \sum_{i=1}^{nc} K_{ci} Q_{ci} \qquad (\$) \tag{1}$$

where

K_p	annual cost per unit of the real power loss ($/kW/year);
K_{ci}	annual cost per unit of the reactive power injection at bus i ($/kVAR/year);
Q_{ci}	reactive power injection at bus i (kVAR);
nc	total number of shunt capacitors to be installed;
$Ploss$	total real power loss (kW).

The total real power loss is defined by

$$Ploss = \underbrace{\sum_{i=1}^{nb} Ploss_i^{(1)}}_{Fund.component} + \underbrace{\sum_{i=1}^{nb} \sum_{h=h_o}^{h_{\max}} Ploss_i^{(h)}}_{Harmonic\ \text{component}} \qquad (\text{kW}). \tag{2}$$

where

nb	number of branches;
h_0	smallest harmonic order of interest;
$h_{\max}$	highest harmonic order of interest.

The fundamental component of the total real power loss is calculated using a three phase power flow algorithm (RDPF) [11].

The harmonic component of the total real power loss is computed by a harmonic power flow algorithm (HPF) [12].

Note that the harmonic component of the total real power loss is small compared with the fundamental one. However, this portion of the total real power loss increases as harmonic-producing loads continue to increase in RDS. Consequently, the undesirable presence of harmonics will cause more equipment overheating, stress on equipment insulation, and equipment failure. Not to mention of course the interference with communication networks. It should be pointed out that the cost of the real power loss per unit is fixed. However, the cost of the reactive power injection per unit varies from one capacitor size to another [1]. Generally, the larger the capacitor size is, the cheaper it becomes.

Constraints: Along with the objective function, there is another significant part of the optimization model that needs to be defined and that is the constraints. In real applications, there are always limits on the choices of control variables. The constraints considered in this research are of two types: equality and inequality.

A. Equality Constraints

The equality constraints are those associated with the nonlinear power flow equations. It is noted in many published papers that the power flow equations are the real and reactive power mismatch equations. The reason for this is that modified versions of conventional power flow programs such as Newton-Raphson method and Gauss Siedel method are widely used. In this work, the power flow representation is based on bus current injections and thus the equality constraints are the bus current mismatch equations. The equality constraints are expressed in a vector form as follows:

$$H(x,u)=0 \tag{3}$$

where:

- x vector of state (dependent) variables;
- u vector of control (independent) variables.

B. Inequality Constraints

The inequality constraints are those associated with the bus voltages, total harmonic distortion levels, and shunt capacitors to be installed.

1) Bus Voltage Limits: The bus voltage magnitudes are to be kept within acceptable operating limits throughout the optimization process

$$V_{\min} \leq |V_i| \leq V_{\max} \tag{4}$$

where

- $V_{\min}$ lower bound of bus voltage limits;
- $V_{\max}$ upper bound of bus voltage limits;
- $|V_i|$ rms value of the ith bus voltage and defined by

$$|V_i| = \sqrt{\left|V_i^{(1)}\right|^2 + \sum_{h=h_0}^{h_{\max}} \left|V_i^{(h)}\right|^2}, \quad i=2,3,\ldots,n \tag{5}$$

where

- n number of buses.

The rms value of the ith bus voltage involves only the fundamental component, the first term of (5), when harmonics are not of interest.

2) Total Harmonic Distortion Limits: The total harmonic distortion at each bus is to be kept less or equal to the maximum allowable harmonic distortion level as shown

$$THD_i(\%) \leq THD_{\max} \tag{6}$$

where

- $THD_{\max}$ maximum allowable harmonic distortion level at each bus.

3) Number and Sizes of Shunt Capacitors: There are constraints associated with the shunt capacitors themselves. Capacitors that are commercially available come in discrete sizes. That is, the shunt capacitors to be dealt with are multiple integers of the smallest capacitor size available

$$Q_{ci} \leq LQ_0, \quad L=1,2,\ldots,nc \tag{7}$$

where

- Q_0 smallest capacitor size available.

It is worth mentioning that the total reactive power injection is not to exceed the total reactive power demand in RDS

$$\sum_{i=1}^{nc} Q_{ci} \leq Q_T \tag{8}$$

where

- Q_T total reactive power demand.

III. Particle Swarm Optimization

PSO is a metaheuristic optimization technique developed in 1995 by Kennedy and Eberhart [13]–[15]. The fundamental idea behind the PSO algorithm is that a population called a swarm is randomly generated. The swarm consists of individuals called particles. Each particle in the swarm represents a potential solution of the optimization problem. Each particle moves through a D-dimensional search space at a random velocity. Each particle updates its velocity and position according to the following equations:

$$\begin{aligned} v_{id}^{k+1} =& wv_{id}^k + c_1r_1\left(pbest_i^k - x_{id}^k\right) \\ &- c_2r_2\left(gbest^k - x_{id}^k\right) \end{aligned} \tag{9}$$

$$x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \tag{10}$$

where

- w inertia weight;
- c_1, c_2 acceleration constants;
- r_1, r_2 two random numbers in the range of [0, 1];
- $pbest_i^k$ best position ever visited by the particle i at the kth iteration;
- $pbest_i^k$ $(pbest_{i1}^k, pbest_{i2}^k, \ldots, pbest_{id}^k, \ldots, pbest_{iD}^k)$;
- $gbest^k$ global best position in the entire swarm;
- $gbest^k$ $(gbest_1^k, gbest_2^k, \ldots, gbest_d^k, \ldots, gbest_D^k)$.

IV. Solution Method

As one of this research objectives, a hybrid particle swam optimization algorithm (HPSO) was developed to find the best combination of the locations and sizes of single phase shunt capacitors in unbalanced radial distribution systems (unbalanced-RDS). The HPSO algorithm combines a discrete version of PSO with a three phase power flow algorithm (RDPF). The former is employed as a global optimizer to optimally locate and rate shunt capacitors, while the latter is utilized to minimize the bus

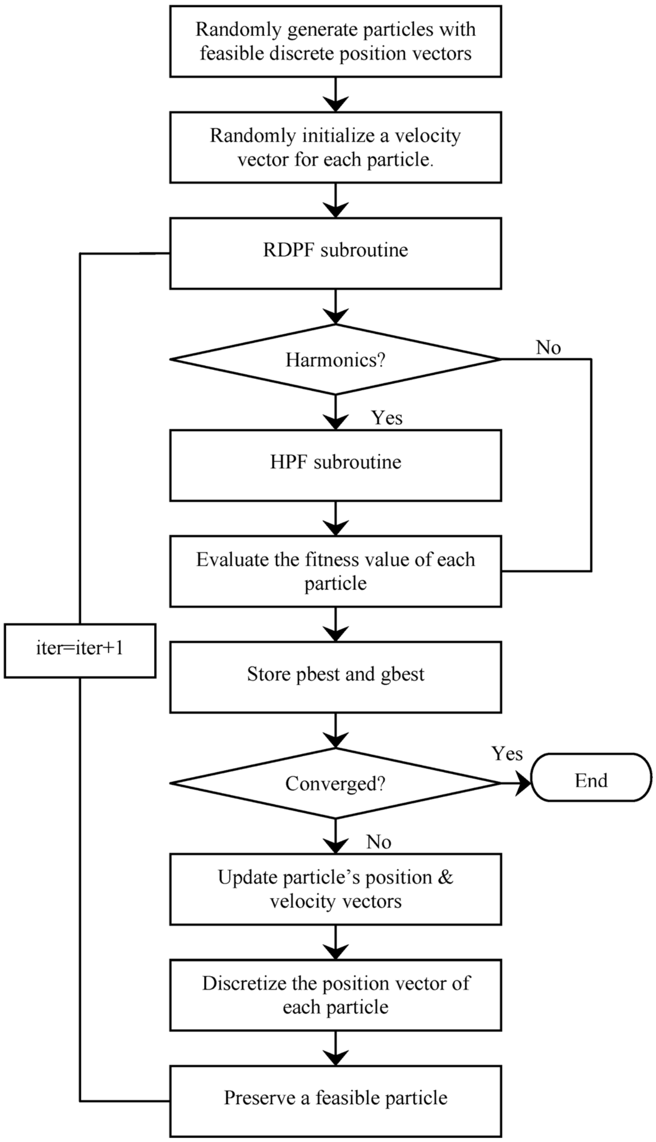

Fig. 1. Flowchart of the HPSO-HP-based algorithm.

current mismatch equations (i.e., the power flow equations). The developed HPSO algorithm starts with generating a swarm of particles randomly in the feasible region of the search space. The feasible swarm is passed to the RDPF subroutine as initial guess to minimize bus current injection mismatch equations. Each particle recalls its best position associated with the best fitness value (i.e., the total cost). Each particle records the best position achieved by the entire swarm. The update process of particles' positions results in continuous values of particles' positions. Thus, discretization of particles' position vectors is made. Once the updated particles' positions are discretized, the particles go through feasibility check to ensure that no particle flies outside the feasible region [16]. When the presence of harmonics is considered, the total harmonic distortion limit at each bus is included as constraints in the optimization problem to ensure that the harmonic distortion levels at all busses are within the allowable limits. A Harmonic power flow (HPF) subroutine is incorporated with the HPSO algorithm to calculate the harmonic bus voltages, harmonic real power losses, and total harmonic distortions. The flowchart of Fig. 1 illustrates the HPSO-HPF-based algorithm that combines the HPF algorithm with the HPSO algorithm.

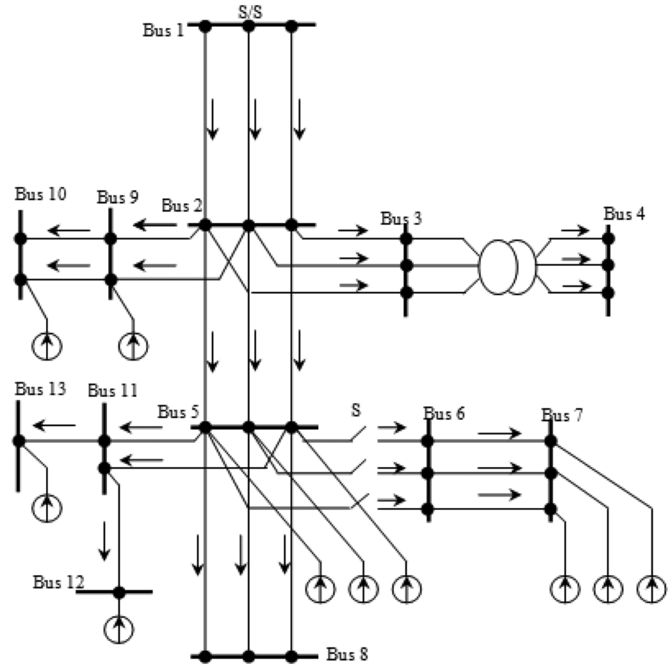

Fig. 2. Unbalanced-13-bus radial distribution system.

V. Results and Discussion

The three algorithms adopted in this work, namely, RDPF, HPF, and PSO, were implemented in MATLAB computing environment on a Dell Laptop with Intel Pentium M processor of 1.86 GHz and RAM of 1 GB. The developed algorithms were tested on an unbalanced-13-bus radial distribution system (unbalanced-13-bus-RDS) whose single line diagram shown in Fig. 2. The unbalanced-13-bus-RDS consists of single, double, and three phase lines and loads. The total real and reactive power demand are 3464.1 kW and 1568.9 kVAR respectively. The system loads are of two types, distributed loads and spot loads. The only supply source in the system is the substation at bus 1. Bus 1 is treated as a slack bus with a constant voltage on each phase of its three phases. The other buses (2–13) are modeled as PQ constant buses. A complete description of the system can be found in [17]. The MVA base value is 10 and the line to line base voltage is the same as the feeder nominal voltage 4.16 kV. The bus voltages are to be kept within 10% of the nominal voltage throughout the optimization process. The cost of real power losses is 168 U.S.$/kW/year, while the cost of the installed capacitor is a function of the capacitor size [1]. Table I lists some commercially available capacitor sizes with their corresponding costs [1].

The number of shunt capacitors to be installed is not to exceed 10 banks of a discrete size of 150 kVAR each. That is to say, the total reactive power injection of these capacitors is not to exceed the total reactive power demand (1568.9 kVAR).

To include the presence of harmonics, the developed HPSO was integrated with a harmonic power flow algorithm (HPF). The proposed (HPSO-HPF) based approach is tested on the

TABLE I
COMMERCIALLY AVAILABLE CAPACITOR SIZES WITH THEIR CORRESPONDING COSTS

Q_c (kVAR)	150	300	450	600	750
K_c ($/kVAR)	0.500	0.350	0.253	0.220	0.276
Q_c (kVAR)	900	1050	1050	1350	1500
K_c ($/kVAR)	0.183	0.228	0.170	0.207	0.201

same test system (13-Bus-RDS). For the distorted voltage-13-Bus-RDS shown in Fig. 2, harmonic-producing loads, namely fluorescent lighting, adjustable speed drives (ASD), and non-specific sources such as PCs, TVs, and etc, are considered. The typical harmonic spectrum of these nonlinear loads is provided in [18]. All loads are treated as constant PQ spot loads for harmonic studies. Load composition in terms of harmonic sources is given in [18].

The developed HPSO-HPF-based approach is applied to find the optimal locations and sizes of shunt capacitors in an unbalanced-IEEE-13-bus radial distribution system (13-Bus-RDS) while taking harmonics into account. The total harmonic distortion levels are to be maintained within 5% of the voltage value as recommended by the IEEE standard 519-1992.

In the presence of harmonics, three different cases are considered to investigate the impact of shunt capacitor installation on the voltage profiles, total harmonic distortions, total real power loss, and net savings.

- **Case 1** represents the system with harmonics consideration before capacitor installation.
- **Case 2** represents the system without harmonics consideration after capacitor installation.
- **Case 3** represents the system with harmonics consideration after capacitor installation.

The PSO parameters were tuned to enhance the performance of the proposed algorithm. For one shunt capacitor to be installed, 20 independent runs were carried out for each PSO parameter. The maximum number of iterations was taken as 50 for the tuning process of each parameter. It was found that the PSO algorithm was less sensitive to its parameters for small problem dimension (the problem dimension was the shunt capacitor location and size). However, the larger the problem dimension is, the more sensitive the PSO algorithm becomes. A swarm size of 20 particles, acceleration constants of 2, and a particle's maximum velocity of 4 were selected. As for the inertia weight (w), it was reduced linearly from 0.9 to 0.4 as recommended in [19].

From the results shown in Table II, installing a shunt capacitor of 600 kVAR at phase c of bus 6 in case 2 will reduce the total real power losses from 192.7494 kW to 179.1373 kW and profit the utility 2,099.694 U.S.$/year. The capacitor size required to bring the violated bus voltages back within the maximum and minimum bus voltage limits are the same for cases 2 and 3, while the PSO-based algorithm selected phase c of bus 5 to be the optimal location of the shunt capacitor in case 3. Before capacitor installation (case 1), the cost of real power losses is 32,326.694 U.S.$/year. In case 2 (after capacitor installation without harmonics consideration), the cost of real power losses is reduced to 30,227 U.S.$/year, while in case 3 (after capacitor installation with harmonics consideration), the cost of real power losses is reduced to 30,271 U.S.$/year. The net savings in case 2 (when harmonics are neglected) are slightly better than those obtained in case 3 (when harmonics are considered). However, the maximum total harmonic distortion level of case 2 ($THD_{\max}^{case2} = 34.8937$) is much higher than that of case 3 ($THD_{\max}^{case3} = 3.1986$).

TABLE II
RESULTS OF THE OPTIMAL PLACEMENT AND SIZING OF ONE SHUNT CAPACITOR IN A 13-BUS-RDS

	Case1	Case2	Case3
Minimum Bus Voltage (p.u)	0.8954	0.9339	0.9343
Maximum Bus Voltage (p.u)	0.9863	0.9993	0.9993
$THD_{\max}$(%)	4.4590	34.8937	3.1986
Reactive Power Injection (kVAR)	-	Q_{6c}=600	Q_{5c}=600
Real Power Losses (kW)	192.7494	179.1373	179.4016
Cost Function ($/year)	32,326.694	30,227	30,271
Net Savings ($/year)	-	2,099.694	2,055,694

The total harmonic distortion reduction in case 3 with respect to case 1 is

$$\begin{aligned} &THD \text{ reduction in case 3 with respect to case 1}\\ &\quad = \frac{\mathrm{THD}_{\max}^{\mathrm{case1}} - THD_{\max}^{case3}}{THD_{\max}^{case1}} \times 100\\ &\quad = \frac{4.4590 - 3.1986}{4.4590} \times 100 = 30.587\%. \end{aligned}$$

The total harmonic distortion reduction in case 3 with respect to case 2 is

$$\begin{aligned} &THD \text{ reduction in case 3 with respect to case 2}\\ &\quad = \frac{\mathrm{THD}_{\max}^{\mathrm{case2}} - THD_{\max}^{case3}}{THD_{\max}^{case2}} \times 100\\ &\quad = \frac{34.8937 - 3.1986}{34.8937} \times 100 = 90.833\%. \end{aligned}$$

The reduction in the maximum total harmonic distortion level in case 3 with respect to cases 1 and 2 justifies the inclusion of harmonics in the optimal capacitor placement and sizing problem.

In Table III, the optimal solution of the HPSO-HPF-based algorithm for the optimal placement and sizing of one shunt capacitor yields voltage profile improvement in both cases (2&3), (see columns 5 and 7 in Table III). However, installing a shunt capacitor without taking harmonics into account (case2) caused a severe harmonic distortion problem (i.e., harmonic distortion levels at all load buses violate the maximum allowable distortion level (5%)), (see column 6 in Table III). In contrast, recognizing the fact that harmonics are propagated throughout distribution systems and including their presence will keep the harmonic distortion levels within the limits (see column 8 in Table III).

The convergence characteristics of the proposed HPSO-HPF-based approach for cases 2 and 3 in the optimal placement and sizing of one shunt capacitor problem with the total cost being

TABLE III
VOLTAGE PROFILES AND TOTAL HARMONIC DISTORTIONS OF A 13-BUS-RDS FOR CASES 1, 2, AND 3 OF THE OPTIMAL PLACEMENT AND SIZING OF ONE SHUNT CAPACITOR

Bus	ϕ	Case1		Case2		Case3	
		$\|V\|$ (p.u)	THD (%)	$\|V\|$ (p.u)	THD (%)	$\|V\|$ (p.u)	THD (%)
1	a	1.0	0	1.0	0	1.0	0
	b	1.0	0	1.0	0	1.0	0
	c	1.0	0	1.0	0	1.0	0
2	a	0.9676	2.3076	0.9638	13.5454	0.9639	1.4876
	b	0.9815	2.1319	0.9688	11.6954	0.9689	1.5940
	c	0.9507	2.3329	0.9919	25.2673	0.9919	0.6851
3	a	0.9647	2.3202	0.9609	13.5890	0.9610	1.4970
	b	0.9795	2.1410	0.9668	11.7210	0.9670	1.6006
	c	0.9480	2.3384	0.9892	25.3245	0.9892	0.6864
4	a	0.9414	2.4064	0.9373	13.9761	0.9374	1.5599
	b	0.9604	2.2155	0.9479	11.9672	0.9480	1.6596
	c	0.9291	2.4003	0.9703	25.8214	0.9704	0.7025
5	a	0.9509	4.4214	0.9413	20.9188	0.9417	2.7861
	b	0.9852	3.6674	0.9598	16.9622	0.9602	2.6385
	c	0.9009	4.6921	0.9837	34.0744	0.9838	1.2107
6	a	0.9509	4.4214	0.9413	20.9188	0.9417	3.0695
	b	0.9852	3.6674	0.9598	16.9622	0.9602	2.8848
	c	0.9009	4.6921	0.9837	34.0744	0.9839	1.9074
7	a	0.9436	4.4940	0.9339	21.2073	0.9343	3.1214
	b	0.9863	3.6541	0.9610	16.9463	0.9614	2.8717
	c	0.8974	4.7189	0.9801	34.2973	0.9803	1.8978
8	a	0.9509	4.4214	0.9413	20.9188	0.9417	3.0695
	b	0.9852	3.6674	0.9598	16.9622	0.9602	2.8848
	c	0.9009	4.6921	0.9837	34.0744	0.9839	1.9074
9	b	0.9702	2.3847	0.9576	12.2656	0.9577	1.8468
	c	0.9542	2.3787	0.9953	25.2876	0.9953	0.7435
10	b	0.9663	2.3351	0.9537	12.2038	0.9539	1.7934
	c	0.9581	2.3551	0.9993	25.1605	0.9993	0.7249
11	a	0.9495	4.5042	0.9398	21.1708	0.9403	3.1420
	c	0.8981	4.7835	0.9808	34.5039	0.9810	1.9491
12	a	0.9442	4.5718	0.9345	21.3886	0.9350	3.1986
13	c	0.8954	4.8590	0.9782	34.8937	0.9784	1.9759

the objective are illustrated in Figs. 3 and 4 respectively. Both Figures indicate the convergence speed of the proposed HPSO-HPF-based solution methodology in finding the global optimal solution of the capacitor allocation and sizing problem.

In order to do more testing on the proposed HPSO-HPF-based algorithm, the capacitor placement and sizing problem is extended to multiple capacitors. Three single phase capacitors are considered instead of one capacitor. The maximum reactive power injection of these capacitors is not to exceed the total reactive demand of the system.

As in the case of one shunt capacitor, the PSO parameters have to be properly adjusted. Taking the total real power loss without harmonic components as an objective, 20 independent runs were conducted to find the best settings of the PSO parameters. 100 iterations were taken as the maximum number of iterations to adjust each of these parameters. A swarm size of 25 particles, acceleration factors of 2 each, and a maximum particle's velocity of 3 were selected.

The developed PSO-based algorithm was able to find the optimal locations and ratings of three shunt capacitors such that the overall cost was minimized. The simulation results are reported in Table IV. Without harmonics consideration (case 2)

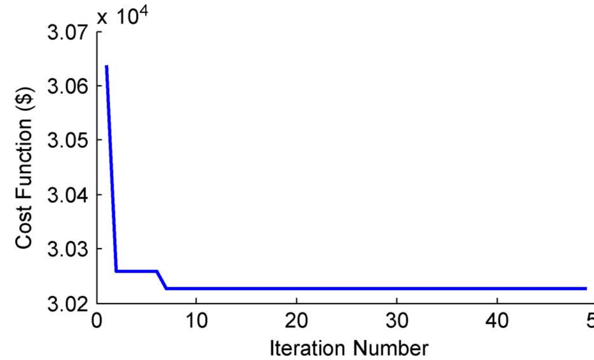

Fig. 3. Convergence characteristics of the HPSO-HPF-based algorithm for case 2 in the optimal placement and sizing of one shunt capacitors.

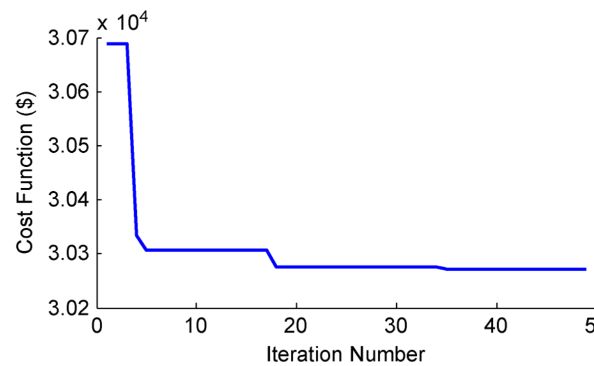

Fig. 4. Convergence characteristics of the HPSO-HPF-based algorithm for case 3 in the optimal placement and sizing of one shunt capacitors.

TABLE IV
RESULTS OF THE OPTIMAL PLACEMENT AND SIZING OF THREE SHUNT CAPACITORS IN A 13-BUS-RDS

	Case1	Case2	Case3
Minimum Bus Voltage (p.u)	0.8954	0.9555	0.9556
Maximum Bus Voltage (p.u)	0.9863	0.9945	0.9947
THD_{max} (%)	4.4590	19.0335	1.7270
Reactive Power Injection (kVAR)	-	Q_{5b}=300 Q_{6a}=450 Q_{6c}=600	Q_{5c}=600 Q_{6a}=450 Q_{6b}=300
Real Power Losses (kW)	192.7494	164.9870	165.2160
Cost Function ($/year)	32,326.694	28,069	28,107
Net Savings ($/year)	-	4,257.694	4,219.694

TABLE V
VOLTAGE PROFILES AND TOTAL HARMONIC DISTORTIONS OF A 13-BUS-RDS FOR CASES 1, 2, AND 3 OF THE OPTIMAL PLACEMENT AND SIZING OF THREE SHUNT CAPACITORS

Bus	ϕ	Case1		Case2		Case3	
		$\lvert V\rvert$ (p.u)	THD(%)	$\lvert V\rvert$ (p.u)	THD(%)	$\lvert V\rvert$ (p.u)	THD(%)
1	a	1.0	0	1.0	0	1.0	0
	b	1.0	0	1.0	0	1.0	0
	c	1.0	0	1.0	0	1.0	0
2	a	0.9676	2.3076	0.9869	1.6192	0.9869	0.5755
	b	0.9815	2.1319	0.9857	8.6309	0.9857	0.6820
	c	0.9507	2.3329	0.9804	8.6917	0.9805	0.4081
3	a	0.9647	2.3202	0.9839	1.6282	0.9839	0.5784
	b	0.9795	2.1410	0.9837	8.6538	0.9837	0.6832
	c	0.9480	2.3384	0.9777	8.7185	0.9777	0.4085
4	a	0.9414	2.4064	0.9606	1.6681	0.9606	0.6046
	b	0.9604	2.2155	0.9647	8.8524	0.9647	0.6986
	c	0.9291	2.4003	0.9589	8.9755	0.9589	0.4164
5	a	0.9509	4.4214	0.9887	6.0031	0.9888	1.4322
	b	0.9852	3.6674	0.9934	19.0335	0.9935	1.4631
	c	0.9009	4.6921	0.9610	8.1086	0.9610	0.7920
6	a	0.9509	4.4214	0.9887	2.2149	0.9889	1.6460
	b	0.9852	3.6674	0.9934	9.4252	0.9935	1.6729
	c	0.9009	4.6921	0.9610	11.3358	0.9611	1.2942
7	a	0.9436	4.4940	0.9814	2.2480	0.9816	1.6665
	b	0.9863	3.6541	0.9945	9.4232	0.9947	1.6583
	c	0.8974	4.7189	0.9574	11.4282	0.9575	1.2701
8	a	0.9509	4.4214	0.9887	2.2149	0.9889	1.6460
	b	0.9852	3.6674	0.9934	9.4252	0.9935	1.6729
	c	0.9009	4.6921	0.9610	11.3358	0.9611	1.2942
9	b	0.9702	2.3847	0.9744	9.1536	0.9744	0.8855
	c	0.9542	2.3787	0.9839	8.5654	0.9839	0.4289
10	b	0.9663	2.3351	0.9705	9.0809	0.9705	0.8363
	c	0.9581	2.3551	0.9878	8.5556	0.9878	0.4206
11	a	0.9495	4.5042	0.9873	2.2368	0.9874	1.6972
	c	0.8981	4.7835	0.9581	11.5182	0.9582	1.3224
12	a	0.9442	4.5718	0.9820	2.2668	0.9822	1.7270
13	c	0.8954	4.8590	0.9555	11.7021	0.9556	1.3398

the developed PSO-based algorithm selected bus 6 as the optimal location for three single-phase capacitors with optimal ratings of 450, 300, and 600 kVAR as shown in Table IV. In case 2, the real power losses were reduced to 164.9870 kW, while in case 3 (with harmonics consideration), the proposed PSO-algorithm selected phase a of bus 5 and phases b and c of bus 6 to be the optimal locations of the three shunt capacitors with the same ratings. It can be observed that the reactive power injections required to minimize the total cost in case 2 (when harmonics are neglected) are equal to the reactive power injections in case 3 (when harmonics are considered). Moreover, the total real power loss in case 3 is higher than that in case 2. As a result, the net savings obtained in case 2 is better than that in case

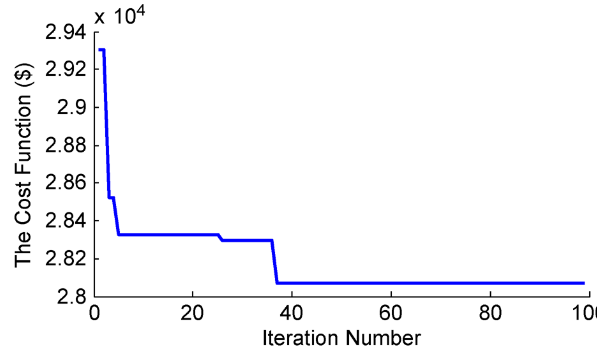

Fig. 5. Convergence characteristics of the HPSO-HPF-based algorithm for case 2 in the optimal placement and sizing of three shunt capacitors.

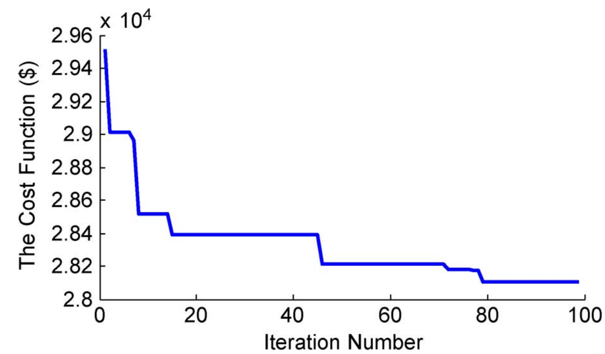

Fig. 6. Convergence characteristics of the HPSO-HPF-based algorithm for case 3 in the optimal placement and sizing of three shunt capacitors.

3. However, the optimal solution in case 3 yields a 61.27% reduction in total harmonic distortion with respect to case 1 and 90.927% reduction in total harmonic distortion with respect to case 2. Consequently, the net savings obtained in case 2 can be justifiably sacrificed to avoid any possible damage to the electric equipment of both the utility and the customers.

Table V demonstrates that the proper installation of three shunt capacitors in the 13-Bus-RDS leads to voltage profile improvement (see columns 5 and 7 in Table V). The harmonic distortion levels at some load buses in case 2, however, exceed the IEEE standard 519-1992 due to neglecting the presence of harmonic sources in the system (column 6 in Table V). In contrast, taking harmonics into account (case 3) maintained the harmonic distortion levels at load buses within the allowable limits $(THD_{\max}(\%) \leq 5)$, (column 8 in Table V). The convergence characteristics of the developed HPSO-HPF-based approach for cases 2 and 3 in the optimal placement and sizing of three shunt capacitors problem with the total cost being the objective are depicted in Figs. 5 and 6.

VI. CONCLUSION

In this paper, the developed HPSO-HPF-based algorithm was tested on an unbalanced 13-bus test system to find the optimal locations and sizes of shunt capacitors taking harmonics into account. The objective was to minimize the total cost of the system real power loss and the shunt capacitors to be installed. The objective function was subject to some operating constraints and power quality constraints. The outcome of this research is that neglecting the presence of harmonics in the system may lead to undesirable harmonic distortion levels causing more damage to the electric equipment of both the electric utility and customers.

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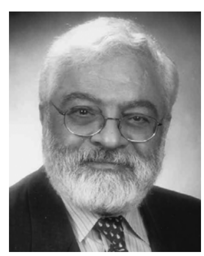

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